

The Time-dependent Supersymmetric Configurations in M-theory and Matrix Models

Bin Chen^{*}

Department of Physics,
Peking University
Beijing 100871, P.R. China

February 1, 2008

Abstract

In this paper, we study the half-supersymmetric time-dependent configurations in M-theory and their matrix models. We find a large class of 11D supergravity solutions, which keeps sixteen supersymmetries. Furthermore, we investigate the isometries of these configurations and show that in general these configurations have no supernumerary supersymmetry. And also we define the matrix models in these backgrounds following Discrete Light-Cone Quantization (DLCQ) prescription.

^{*}email:bchen01@pku.edu.cn

1 Introduction

It is very important in string theory to understand the mysterious cosmological singularity. From string theory's point of view, such singularity should be resolved by the stringy effects. It is widely believed that near the big-bang/crunch singularity the usual concept of spacetime in General Relativity would not be applicable and there should be some new concepts and ideas to save the life. Very recently several proposals have been raised to address the issue[1, 2].

One of the remarkable proposal is the idea of matrix big bang[1]. The authors started from IIA string theory in a linear null dilaton background. Such a background does not change the dimension of the spacetime and keep one-half of the original supersymmetries. And the perturbative string theory in the background is exactly solvable in the sense that all the vertex operators and amplitudes could be defined formally. However, due to the linear null dilaton the string coupling becomes very strong near the big-bang (or big-crunch) singularity and the perturbative string theory is ill-defined there. A dual matrix string theory has been proposed and the physics near the big bang is described by a weakly coupled two-dimensional Yang-Mills theory. The matrix degrees of freedom, instead of point-particles or strings, are used to describe the physics near the singularity.

In [3], it has been pointed out that there exist other supersymmetric backgrounds, in which one may define cosmological matrix models. And in [4], the matrix string theory in the PP-wave metric with a linear null dilaton has been studied. It would be interesting to search for other classes of time-dependent supersymmetric configurations. Due to the existence of the supersymmetries, the matrix models in these backgrounds could be stable. However, it is remarkable that the standard sixteen supersymmetries, which are characterized by the Killing spinors satisfying $\Gamma^+ \epsilon = 0$, could only be realized nonlinearly in the matrix models and does not guarantee that the corresponding matrix models are supersymmetric[1, 4]. This inspires us to look for the configurations with the supernumerary supersymmetries, which may allow linearly realized supersymmetries in their matrix models.

In section 2, we will try to find the time-dependent supersymmetric solutions of 11D supergravity. We obtain a large class of configurations (28), which are akin to the usual plane-wave geometries. Such configurations keep at least one-half of the original supersymmetries, characterized by the Killing spinors satisfying $\Gamma^+ \epsilon = 0$.

In section 3, in order to find out the extra supersymmetries and understand the isometric symmetries, we investigate the Killing vectors of the configurations found in section 2. We manage to figure out the configurations which could have extra supersymmetries. Such configurations should have the Killing vector with nonvanishing ∂_u component and being independent on v . However, it turns out that all these configurations can be transformed to the well-studied smooth homogeneous plane-waves with supernumerary supersymmetries, including the maximal Cahen-Wallach metric and its generalization[5, 6]. This leads us to conclude that in general the metric (28) can only keep the standard sixteen supersymmetries.

In section 4, we formulate the matrix models in these backgrounds following the DLCQ

prescription. In particular, we focus on the backgrounds with the metrics being exponential functions. Such backgrounds have better isometries and are quite similar to the one studied in [4].

2 The 1/2 BPS configurations in 11d Supergravity

Let us start from the equations of motions of 11-dimensional supergravity

$$R_{MN} = \frac{1}{12}(F_{MPQR}F_N{}^{PQR} - \frac{1}{12}g_{MN}F^2) \quad (1)$$

$$d * F = \frac{1}{2}F \wedge F \quad (2)$$

and the Killing spinor equations

$$\tilde{D}_M\epsilon = (D_M - \Omega_M)\epsilon \quad (3)$$

where D_M is the spin connection defined by

$$D_M = \partial_M + \frac{1}{4}\omega^{ab}\Gamma_{ab} \quad (4)$$

and

$$\Omega_M = \frac{1}{288}F_{PQRS}(\Gamma^{PQRS}{}_M + 8\Gamma^{PQR}\delta_M^S). \quad (5)$$

We would like to find the solutions to the equations of motions, which have at least sixteen standard supersymmetries, corresponding to the Killing spinors ϵ satisfying $\Gamma^+\epsilon = 0$.

Let us firstly consider the solutions to the vacuum equations of motions. Inspired by the recent studies on the time-dependent backgrounds in string theory and M-theory[1, 3], we make the following ansatz:

$$ds^2 = 2A_0(u)dudv + A_i(u)(dx^i)^2, \quad (6)$$

where the $A_0(u), A_i(u), i = 1, \dots, 9$ are the functions of u . For the metrics which are not diagonal in the i, j -directions but are the functions of u , they could be diagonalized into (6).

An orthogonal frame is

$$\begin{aligned} \theta^+ &= \sqrt{A_0(u)}du \\ \theta^- &= \sqrt{A_0(u)}dv \\ \theta^I &= \sqrt{A_i(u)}dx^i\delta_i^I. \end{aligned} \quad (7)$$

The nonvanishing spin connections are

$$\omega^{-i} = -\frac{\partial_u \sqrt{A_i}}{\sqrt{A_0}}dx^i, \quad \omega^{-+} = -\frac{\partial_u \sqrt{A_0}}{\sqrt{A_0}}du \quad (8)$$

and the only non-zero Ricci tensor is

$$R_{uu} = \sum_i \frac{\sqrt{A_0}}{\sqrt{A_i}} \left(-\partial_u \left(\frac{\partial_u \sqrt{A_i}}{\sqrt{A_0}} \right) + \frac{\partial_u \sqrt{A_i} \partial_u \sqrt{A_0}}{A_0} \right). \quad (9)$$

Therefore we have just one equation of motion for the functions A_0, A_i 's. Next let us check the remaining supersymmetries. In the vacuum case, $\Omega_M = 0$ and the Killing spinor equations are

$$\begin{aligned} \partial_u \epsilon &= -\frac{1}{2} (\omega_u^{+-} + \omega_u^{-+} \Gamma_{-+} + \omega_u^{-i} \Gamma_{-i}) \epsilon \\ \partial_i \epsilon &= -\frac{1}{2} \omega_i^{-i} \Gamma_{-k} \epsilon. \end{aligned} \quad (10)$$

Choosing a constant spinor ϵ_0 with $\Gamma^+ \epsilon_0 = 0$, then the Killing spinor satisfying the above equations is

$$\epsilon(u) = \exp\left(-\frac{1}{2} \int \omega_u^{-+} du\right) \epsilon_0. \quad (11)$$

Thus the metric (6) keeps at least one-half of the original supersymmetries. To search for the possible extra supersymmetries, one has to look for the Killing spinor with $\Gamma^+ \epsilon \neq 0$.

Another class of well-studied supersymmetric configurations in 11D supergravity is the plane-wave geometries, in the presence of a constant 4-form field strength. Among them, the Cohen-Wallash metric is maximal supersymmetric[8] and the homogeneous plane-waves have supernumerary supersymmetries. This inspires us to look for the plane-wave-like metrics which keep at least one-half supersymmetries, besides the metric (6).

Let us introduce a constant 4-form field strength

$$F_{u123} = f_0(u), \quad (12)$$

and make the following ansatz on the metric

$$ds^2 = 2A_0(u)dudv + B_0(u, x)du^2 + A_i(u)(dx^i)^2 + B_i(u, x)dx^i du, \quad (13)$$

where the B_0, B_i 's are the functions of u and x . Temporarily we do not assume that the B_0 is quadratic in x and the B_i 's are linear in x so that the metric is akin to the plane-wave metric. Later on, we will show that the supersymmetric condition requires (24) and then the equation of motion leads to (27).

The metric (13) allows an orthogonal frame

$$\theta^+ = \sqrt{A_0(u)} du \quad (14)$$

$$\theta^- = \sqrt{A_0(u)} dv + \frac{B_0(u, x)}{2\sqrt{A_0(u)}} du + \frac{B_i(u, x)}{2\sqrt{A_0(u)}} dx^i \quad (15)$$

$$\theta^I = \sqrt{A_i(u)} dx^i \delta_i^I. \quad (16)$$

The corresponding spin connections are

$$\begin{aligned}
\omega^{-+} &= -\frac{\partial_u \sqrt{A_0}}{\sqrt{A_0}} du \\
\omega^{+i} &= 0 \\
\omega^{ij} &= -\frac{\partial_i B_j}{2\sqrt{A_i A_j}} du \\
\omega^{-i} &= \frac{1}{\sqrt{A_i}} \left(\frac{\partial_i B_0}{2\sqrt{A_0}} - \frac{\partial_u B_i}{2\sqrt{A_0}} + \frac{\partial_u \sqrt{A_0}}{A_0} B_i \right) du \\
&\quad - \frac{\partial_u \sqrt{A_i}}{\sqrt{A_0}} dx^i + \sum_{j \neq i} \frac{\partial_i B_j}{2\sqrt{A_0 A_i}} dx^j.
\end{aligned} \tag{17}$$

Before we give the Ricci tensor and solve the equations of motions, let us turn to the Killing spinor equations first. With the constant field strength (12), we have

$$\Omega_v = 0 \tag{18}$$

$$\Omega_u = -\frac{1}{12}(\Gamma^{+-123} + \Gamma^{123}) \frac{f_0}{\sqrt{A_1 A_2 A_3}} \tag{19}$$

$$\Omega_i = \frac{1}{24}(3\Gamma^{123}\Gamma^i + \Gamma^i\Gamma^{123})\Gamma^+ \frac{\sqrt{A_i}f_0}{\sqrt{A_0 A_1 A_2 A_3}}, \tag{20}$$

and the Killing spinor equations

$$\begin{aligned}
\partial_v \epsilon &= 0 \\
\partial_u \epsilon &= \Omega_u \epsilon - \frac{1}{2}(\omega_u^{-+} + \omega_u^{+-}\Gamma^{-+} + \omega_u^{-i}\Gamma^{+i} + \omega_u^{ij}\Gamma^{ij})\epsilon \\
\partial_i \epsilon &= \Omega_i \epsilon - \frac{1}{2}\omega_i^{-k}\Gamma^{+k}.
\end{aligned} \tag{21}$$

The first equation tell us that the ϵ is independent of v and if one requires it to be annihilated by Γ^+ , it is also independent of x^i . In other words, introducing a ϵ_0 with $\Gamma^+ \epsilon_0 = 0$, we have

$$\epsilon(u) = \exp(x\Gamma^{123} + y + z\Gamma^{ij})\epsilon_0 \tag{22}$$

with

$$x = -\frac{1}{4} \int \frac{f_0}{\sqrt{A_1 A_2 A_3}} du, \quad y = \frac{1}{4} \ln A_0, \quad z = \int \frac{\partial_i B_j}{4\sqrt{A_j}} du. \tag{23}$$

And since $\partial_i \epsilon = 0$, one has to require $\partial_i B_j$ is independent of x^i . This help us to fix

$$B_i = A_{ij}(u)x^j \tag{24}$$

with $A_{ij} = -A_{ji}$ ¹, in order to keep one-half of the supersymmetries.

¹The antisymmetric condition is not necessary. We would like to thank the authors of [13] to point this out. For some more general BPS configuration, see [13].

With the choice (24), the Ricci tensor has the nonvanishing component

$$R_{uu} = \sum_i \frac{\sqrt{A_0}}{\sqrt{A_i}} \left(-\partial_u \left(\frac{\partial_u \sqrt{A_i}}{\sqrt{A_0}} \right) - \frac{1}{2\sqrt{A_0 A_i}} \partial_i^2 B_0 + \frac{\partial_u \sqrt{A_0} \partial_u \sqrt{A_i}}{A_0} + \sum_{j \neq i} \frac{A_{ij}^2}{4A_j \sqrt{A_0 A_i}} \right). \quad (25)$$

The nontrivial equation of motion is

$$R_{uu} = \frac{f_0^2}{2A_1 A_2 A_3} \quad (26)$$

The right-hand side is completely independent of x^i , and in R_{uu} the only trouble comes from the term involving B_0 . A natural choice is to let B_0 be bilinear in x^i , namely

$$B_0 = B_{ij}(u) x^i x^j. \quad (27)$$

In the following, we will work with (24,27). Now the metric (13) looks quite similar to the homogeneous plane waves discussed in [5] but due to the dependence on u , the metric (13) is in general not homogeneous.

Therefore, we find a class of time-dependent supersymmetric configurations akin to the plane-waves

$$ds^2 = 2A_0(u)dudv + B_{ij}(u)x^i x^j (du)^2 + A_i(u)(dx^i)^2 + A_{ij}(u)x^j dx^i du, \quad (28)$$

One may take (6) as a special sub-class of (28).

3 Killing vectors and Supernumerary supersymmetries

We have shown that the configurations (28) are at least half-supersymmetric in M-theory. One interesting question is that are there extra supersymmetries in these configurations? One may address this issue directly by working with the Killing spinor equations. Here we take another approach by checking if such configurations have Killing vector with nonvanishing ∂_u component and no dependence on v . The point is that if the Killing vector has nonvanishing ∂_u component, the corresponding configurations may have extra supersymmetries. The advantage of this approach is that it allows us to determine the isometries of the configurations.

A Killing vector, which is of the form

$$K = K^u \partial_u + K^v \partial_v + K^i \partial_i, \quad (29)$$

satisfies the equation

$$L_K g_{AB} = K^C \partial_C g_{AB} + \partial_A K^C g_{CB} + \partial_B K^C g_{CA} = 0. \quad (30)$$

Without losing generality, we set A_{ij} in the metric (28) to be zero. Then from the Killing equation, we have a set of relations:

$$0 = K^u \partial_u B_0 + K^i \partial_i B_0 + 2\partial_u K^u B_0 + 2\partial_u K^v A_0 \quad (31)$$

$$0 = \partial_v K^u \quad (32)$$

$$0 = K^u \partial_u A_0 + \partial_u K^u A_0 + \partial_m K^m A_0 \quad (33)$$

$$0 = B_0 \partial_i K^u + \partial_i K^v A_0 + \partial_u K^i A_i \quad (34)$$

$$0 = \partial_v K^i A_i + \partial_i K^u A_0 \quad (35)$$

$$0 = (K^u \partial_u A_i) \delta_{ij} + \partial_i K^j A_j + \partial_j K^i A_i \quad (36)$$

From (32), we know that K^u is independent of v and then from (33), we find that K^v is at most to be linear in v . And from acting ∂_v on (36) and (35), we see that the K^u is at most the linear function of x^i , namely

$$K^u = \sum_i a_i(u) x^i + a_0(u). \quad (37)$$

Then from (33), we get

$$K^v = -\frac{\sum_i \partial_u(a_i A_0)x^i + \partial_u(a_0 A_0)}{A_0} v + h(u, x^i) \quad (38)$$

where $h(u, x^i)$ is a function to be determined. From (35), we have

$$K^i = -\frac{a_i A_0}{A_i} v + e^i(u, x^i) \quad (39)$$

where e^i is only the function of u, x^i . And from (34), we find that

$$a_i = \frac{\sqrt{c_i A_i}}{A_0} \quad (40)$$

$$e^i = -\int \frac{a_i B_0 + \partial_i h A_0}{A_i} du + f^i(x^i). \quad (41)$$

Next, expanding K^i in powers of x^k

$$K^i = -\frac{a_i A_0}{A_i} v + z_i(u) + z_{ik}(u)x^k + \frac{1}{2}z_{ikl}(u)x^k x^l + \frac{1}{3!}z_{iklm}(u)x^k x^l x^m + \dots \quad (42)$$

and from (36), we find that the terms higher than quadratic vanish and $z_{ijk} = 0$, if j or $k \neq i$. The only nonvanishing coefficients are

$$z_{iil} = z_{ili} = -\frac{\partial_u A_i}{2A_i} a_l, \quad z_{ii} = -\frac{a_0 \partial_u A_i}{2A_i} \quad (43)$$

and z_{ij} 's, which satisfy the relation

$$A_j z_{ji} + A_i z_{ij} = 0, \quad \text{if } i \neq j. \quad (44)$$

Furthermore, taking into account of the quadratic form of B_0 and the relation (31), we know that a_i has to be vanishing

$$a_i = 0. \quad (45)$$

From the linear term in v , quadratic terms vx^j in (31), we get the following relations

$$a_0(u) = \alpha \frac{\int A_0 du}{A_0} + \beta, \quad (46)$$

$$\partial_u h_0(u) = 0, \quad (47)$$

$$h_{ii}(u) = \frac{A_i}{A_0} \partial_u \left(\frac{a_0 \partial_u A_i}{2A_i} \right), \quad (48)$$

$$\partial_u h_j = \sum_i \frac{B_{ji}}{A_0} \int \frac{h_i A_0}{A_i} du, \quad (49)$$

where α, β are constants and h_0, h_i, h_{ii} are the expansive coefficients of the function h in terms of x :

$$h(u, x^i) = h_0(u) + h_i(u)x^i + \frac{1}{2}h_{ij}x^i x^j + \dots \quad (50)$$

And finally from the quadratic $x^j x^k$ terms in (31), we get

$$0 = a_0 \partial_u B_{ii} - \frac{a_0 \partial_u A_i}{A_i} B_{ii} + 2\partial_u a_0 B_{ii} + 2 \sum_j f_{ji} B_{ji} + 2\partial_u h_{ii} A_0, \quad (51)$$

$$0 = a_0 \partial_u B_{jk} + 2\partial_u a_0 B_{jk} - \frac{a_0 \partial_u A_j}{A_j} B_{jk} + 2 \sum_i f_{ik} B_{ij}, \quad j \neq k \quad (52)$$

with the constants f_{ij} 's constrained by

$$A_j f_{ji} + A_i f_{ij} = 0. \quad (53)$$

Then the components of the Killing vector read

$$K^u = a_0(u), \quad (54)$$

$$K^v = -\frac{\partial_u(a_0 A_0)}{A_0} v + h_0(u) + h_i(u)x^i + h_{ii}(u)(x^i)^2, \quad (55)$$

$$K^i = -\int \frac{h_i A_0}{A_i} du - \frac{a_0 \partial_u A_i}{2A_i} x^i + f_{ik} x^k \quad (56)$$

From the constraint (53) on the constants f_{ij} 's, we know that if $A_i(u)$ is not proportional to $A_j(u)$, $f_{ij} = 0$. This indicates that the usual rotational Killing symmetry $x^i \partial_j - x^j \partial_i$ gets lost in this case.

Since either f_{ij} being antisymmetric or being zero, the relevant term $\sum_j f_{ji} B_{ji}$ in (51) is vanishing and (51) gives us a relation between a_0 and B_{ii} once the form of the metric is fixed. In order to have nonvanishing K^u , the form of the metric is highly constrained by (51,52).

The cases when the Killing vector has nonvanishing K^u component are interesting since the corresponding configurations may have extra supersymmetries. This could be seen as follows[6]. Consider the Killing vector

$$K = \bar{\epsilon} \Gamma^\mu \epsilon \partial_\mu, \quad (57)$$

which has the component $K^u = \frac{1}{\sqrt{2}}(\Gamma^+ \epsilon)^T (\Gamma^+ \epsilon)$. For the supersymmetric configurations we discussed above, we always have sixteen standard supersymmetries, characterized by the Killing spinors satisfying $\Gamma^+ \epsilon = 0$. These Killing spinors can not give us the nonvanishing K^u . However, the extra supersymmetries with $\Gamma^+ \epsilon \neq 0$ will give us the nonvanishing Killing component K^u . On the other hand, the metric with a nonvanishing Killing vector K^u could have supernumerary supersymmetries.

Let us figure out in which case we can have the Killing vector with a nonvanishing K^u . Firstly let us consider the case when $B_0 = 0$. From (51) we know that g_{ii} should be constant. Only in very special situation, say all $A_i \propto A_0$ and A_i 's being the exponential function of u^2 , one has a nonvanishing constant g_{ii} . Generically $g_{ii} = 0$, which require

- $a_0 = 0$;
- A_i 's being constant with no restriction on a_0 ;
- A_i 's being exponential function with a_0 being constant.

Moreover, from $\partial_u h_j = 0$ we get $h_j = \alpha_j u + \beta_j$ with α_j, β_j being constant.

Let us see a few examples:

- The trivial example is that all the A_0, A_i 's are constant, which could be normalized to be unit. Obviously, all the discussions above reduce to the study of the Killing vectors in the flat spacetime;
- The first nontrivial example is the configuration corresponding to the null linear dilaton background[1]:

$$ds^2 = e^{2Qu/3} ds_{10}^2 + e^{-4Qu/3} (dx^{11})^2 \quad (58)$$

where Q is a constant. In this case, the requirement that h_{99} is constant lead to $a_0 = \beta$ and $h_{ii} = 0$. And since $A_i = A_0$, for $i = 1, \dots, 8$, one has rotational symmetry among x^i 's and also a rotational symmetry between u and x^i .

- A more general case with vanishing B_0 is that all the $A_i(u)$'s are exponential functions but different so that $f_{ij} = 0$. The components of the Killing vectors could be of the form:

$$\begin{aligned} K^u &= \beta \\ K^v &= -\beta(\partial_u \ln A_0)v + h_0(u) + \sum_i h_i(u)x^i \\ K^i &= -\int \frac{h_i A_0}{A_i} du - \frac{\beta}{2}(\partial_u \ln A_i)x^i \end{aligned} \quad (59)$$

Next we turn to the more general metrics with a nonvanishing B_0 . From (51), we know that it is hard to find nontrivial solution when $h_{ii} \neq 0$ and generically we have $a_0 = 0$. A case with a nonvanishing a_0 is that when $A_i(u)$ take the exponential forms $A_i(u) = \exp(\gamma_i u)$ and $a_0 = \beta$,

$$B_{ii} = c_i A_i \quad (60)$$

with c_i being constants. One may set $\beta = 1$ and then

$$B_{jk} = e^{\gamma_j u} (e^{2uf} B_s e^{-2uf})_{jk} \quad (61)$$

where B_s is a constant symmetric matrix. It is always possible to arrange the matrix f_{ij} so that the matrix B_{jk} takes a block form. In other words, if $A_j \neq A_k$, $B_{jk} = 0$.

Another special case is when all A_i are constants and then one may has $a_0 = u$, and

$$B = \frac{1}{u^2} e^{2f \log u} B_s e^{-2f \log u}. \quad (62)$$

Usually, a Killing vector with nonvanishing K^u component is not enough to ensure the extra supersymmetries. Since in our discussion the Killing spinor is independent of v , the corresponding Killing vector cannot have v dependence. From the form of K^v , the condition to have no term proportional to v is that $A_0 = \text{constant}$ which could be set to 1.

From the above discussion we learn that the nonvanishing K^u gives the strong constraint on the form of the metric. In the case with $B_0 = 0$, we know that when A_0, A_i are exponential functions, we have $K^u = \beta$ which could be chosen to be 1 for simplicity. And one need to take $A_0 = 1$ so that $K^v = 0$. But we always has a nonvanishing term $-\frac{\gamma_i}{2} x^i$ in K^i so the Killing vector is

$$K = \partial_u - \frac{\gamma_i}{2} x^i \partial_i. \quad (63)$$

after choosing h_i 's vanishing. In the cases discussed in [1, 3], A_0, A_i 's take the exponential forms, but since $A_0 \neq 1$, the background cannot have the supernumerary supersymmetry.

In the case with nonvanishing B_0 , the existence of the nonvanishing K^u requires B_{jk} take the form (61) or (62). In the latter case, the existence of nonvanishing K^v shows that there is no extra supersymmetry. On the other hand, in the case (61), there may exist the extra supersymmetries when $A_0 = 1$.

At this moment, let us do a short summary. The time-dependent supersymmetric configurations with the possible supernumerary supersymmetries are

- $B_0 = 0, A_0 = 1$, and A_i 's are exponential functions;
- $B_0 \neq 0, A_0 = 1$, A_i 's are exponential functions and B_{jk} take the form of (61).

So let us focus on the metrics of the form

$$ds^2 = 2dudv + \sum_i e^{\gamma_i u} (dx^i)^2 + \sum_{ij} B_{ij} x^i x^j (du)^2. \quad (64)$$

However, it is easy to see that the above metrics could be transformed to the well-studied ones by changing coordinates. In fact, after defining

$$\begin{aligned}\tilde{x}^i &= e^{\gamma_i u/2} x^i \\ \tilde{v} &= v - \sum_i \frac{\gamma_i}{4} e^{\gamma_i u} (x^i)^2,\end{aligned}\tag{65}$$

the above metric takes a form

$$ds^2 = 2dud\tilde{v} + (d\tilde{x}^i)^2 + \sum_i \frac{\gamma_i^2}{4} (\tilde{x}^i)^2 (du)^2 + \sum_{ij} \tilde{B}_{ij} \tilde{x}^i \tilde{x}^j (du)^2\tag{66}$$

where

$$\tilde{B}_{ij} = (e^{2uf} B_s e^{-2uf})_{ij}.\tag{67}$$

On the other hand, in the new coordinates $(u, \tilde{v}, \tilde{x}^i)$, the 4-form field strength is

$$F_{u\tilde{1}\tilde{2}\tilde{3}} = \text{constant}.\tag{68}$$

Therefore, in terms of the new coordinates the background (64) with the possible supernumerary supersymmetries is reduced to the well-studied homogeneous plane-waves[6].

We conclude that our time-dependent supersymmetric configurations of the metric form (28) have no supernumerary supersymmetry, except the cases with constant A_0, A_i 's and the appropriately chosen B_{ij}, A_{ij} 's.

4 Matrix models

In general, the definition of the matrix models in the curved backgrounds and the time-dependent backgrounds is a subtle issue. In our case, we are not be able to derive the matrix models following the argument in [10, 11]. Instead, we obtain the matrix models by considering the lightcone gauge action of a single massless particle in eleven dimensions with momentum $p_- = N/R$ [12]. The bosonic action of the matrix model in a curved background is

$$\begin{aligned}S_B &= \int d\tau STr \left(\frac{1}{2R} (g^{uv}) g_{ij} D_\tau X^i D_\tau X^j - \frac{1}{R} g_{ij} g^{vi} D_\tau X^j + \frac{1}{2g^{uv} R} (g_{ij} g^{vi} g^{vj} - g^{vv}) \right. \\ &\quad \left. + \frac{R}{4g^{uv}} g_{ik} g_{jl} [X^i, X^j] [X^k, X^l] - \frac{i}{2} A_{uij} X^i X^j \right)\end{aligned}\tag{69}$$

and the fermionic action is of the form

$$S_F = \int d\tau STr \left(i\psi^T D_\tau \psi - \frac{1}{R} g_{ij} \psi^T \Gamma^i [X^j, \psi] + \frac{i}{48R} \psi^T \Gamma^{ijk} \psi F_{uijk} + \frac{1}{16R} \psi^T \Gamma^{ij} \psi \partial_j (g_{+i}) \right)\tag{70}$$

Note that in the above relation we use a loose notation where the index in Γ is not in the frame.

We have seen that the metric (28) with A_0, B_0, A_i 's being exponential have better geometric property from the discussion on the Killing vector. Let

$$A_0 = e^{\gamma_0 u}, \quad A_i = e^{\gamma_i u}, \quad B_0 = \sum_i c_i e^{\gamma_i u} (x^i)^2, \quad (71)$$

where c_i 's being constant, and for simplicity, we let B_0 to be diagonal. From the equation of motion, we have

$$A_{ij} = A_{ij}^0 e^{(\gamma_i + \gamma_j)u/2}, \quad F_{uijk} = e^{\frac{(\gamma_i + \gamma_j + \gamma_k)}{2}u} \epsilon_{ijk} f^0 \quad (72)$$

where $A_{ij}^0 = -A_{ji}^0 = \text{constant}$, f^0 being constant.

With respect to the metric (28) and (71, 72), the action of the matrix models is

$$\begin{aligned} S_B &= \int d\tau STr \left(\frac{1}{2R} \sum_i e^{(\gamma_i - \gamma_0)\tau} (D_\tau X^i)^2 + \frac{1}{2R} \sum_{ij} e^{(\frac{\gamma_i + \gamma_j}{2} - \gamma_0)\tau} A_{ij}^0 X^j D_\tau X^i + \frac{1}{2R} \sum_i e^{(\gamma_i - \gamma_0)\tau} c_i (X^i)^2 \right. \\ &\quad \left. + \frac{R}{4} \sum_{ij} e^{(\gamma_0 + \gamma_i + \gamma_j)\tau} [X^i, X^j]^2 + \frac{if^0}{2} e^{\frac{(\gamma_i + \gamma_j + \gamma_k)}{2}\tau} \epsilon_{ijk} X^i X^j X^k \right) \\ S_F &= \int d\tau STr \left(i\psi^T D_\tau \psi - \frac{1}{R} \sum_i e^{\frac{\gamma_i}{2}\tau} \psi^T \Gamma^i [X^i, \psi] + \frac{if^0}{8R} \psi^T \Gamma^{ijk} \psi + \sum_{ij} \frac{A_{ij}^0}{16R} \psi^T \Gamma^{ij} \psi \right) \end{aligned} \quad (73)$$

Although we have formally defined the matrix models in the time-dependent supersymmetric backgrounds, we are not certain if such a definition make sense or not. The point is that the action of the matrix model (69, 70) is defined in a *weakly* curved background, which require the background deviate from the flat spacetime not very much. However, in our cases, the metric dependence on u is of exponential form such that the background could be far from flat. Especially, near the big-bang or big-crunch singularity, the metric could be singular. One may needs to check the consistency of the action. In [4], a matrix string theory action in a null linear dilaton PP-wave background has been proposed and was argued to truly describe strongly coupled string theory. Due to the close relation between the matrix string and the matrix model and the similarity between our background and the one in [4], we are inclined to believe that the action (73) make sense.

It would be interesting to study the physical properties of these matrix models and investigate their possible cosmological applications. Also it could be important to check the stability of these backgrounds.

Acknowledgement

We would like to thank Miao Li for reading the manuscript and valuable suggestions. We are grateful to Interdisciplinary Center for Theoretical Studies, where the work was finished. The work was supported by CNSF Grant No. 10405028 and the Key Grant Project of Chinese Ministry of Education (NO. 305001).

References

- [1] B. Craps, S. Sethi and E. Verlinde, *A Matrix Big Bang*, hep-th/0506180.
- [2] J. McGreevy, E. Silverstein, *The Tachyon at the End of the Universe*, hep-th/0506130.
- [3] Miao Li, *A class of cosmological matrix models*, hep-th/0506260.
Miao Li and Wei Song, *Shock waves and cosmological matrix models*, hep-th/0507185.
- [4] S.R. Das and J. Michelson, *pp Wave Big Bangs: Matrix Strings and Shrinking Fuzzy Spheres*, hep-th/0508068.
- [5] M. Blau and M. O'Loughlin, *Homogeneous Plane Waves*, Nucl.Phys.B654 (2003)135-176, hep-th/0212135.
- [6] M. Blau, P. Meessen and M. O'Loughlin, *Goedel, Penrose, anti-Mach: extra supersymmetries of time-dependent plane waves*, JHEP 0309 (2003) 072, hep-th/0306161.
- [7] J. P. Gauntlett, and S. Pakis, *The Geometry of D=11 Killing Spinors*, JHEP 0304 (2003) 039, hep-th/0212008.
J. P. Gauntlett, J. B. Gutowski and S. Pakis, *The Geometry of D=11 Null Killing Spinors*, JHEP 0312 (2003) 049, hep-th/0311112.
- [8] J. Figueroa-O'Farrill and G. Papadopoulos, *Homogeneous fluxes, branes and a maximally supersymmetric solution of M-theory*, JHEP 0108 (2001) 036, hep-th/0105308; P.T. Chrusciel and J. Kowalski-Glikman, *The Isometry Group And Killing Spinors For The PP Wave Space-Time In D = 11 Supergravity*, Phys.Lett. B149 (1984)107.
- [9] J. Figueroa-O'Farrill and G. Papadopoulos, *Maximally supersymmetric solutions of ten- and eleven-dimensional supergravities*, JHEP 0303 (2003) 048, hep-th/0211089.
- [10] A. Sen, *D0 Branes on T^n and Matrix Theory*, Adv. Theor. Math. Phy. **2**(1998)51-59, hep-th/9709220.
- [11] N. Seiberg, *Why is the Matrix Model Correct?*, Phys. Rev. Lett. **79**(1997)3577-3580, hep-th/9710009.
- [12] W. Taylor and M. Van Raamsdonk, *Multiple D0-branes in weakly curved backgrounds*, Nucl. Phys. **B558**(1999)63, hep-th/9904095; *Multiple Dp-branes in Weak Background Fields*, Nucl. Phys. **B573**(2000)703, hep-th/9910052.
- [13] H. Kodama and N. Ohta, *Time-dependent Supersymmetric Solutions in M Theory and Compactification-Decompactification Transition*, hep-th/0605179.